Balancing Sustainability and Output in Renewable-Resource Differential Games Via a Fairness-Competition Lever

Yi Han, Wenjun Mei

Peking University

北京大學
PEKING UNIVERSITY

IFAC 2026
BUSAN

Common pool resource problem

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Renewable stock · Shared access · Subtractive extraction

Interventions with specific instruments

taxes · quotas · access restrictions · subsidies

Governance principles

competition-oriented incentives ↔ pooling-oriented sharing

Question: how does a fairness–competition orientation of a rule shape strategic harvesting behavior?

Model Setup

- Players: $i \in \{1,2\}$
- Resource stock: $x(t) \in [0,K]$, where K is the carrying capacity.
- Action: Player i chooses extraction effort $e_i(t) \geq 0$.
- Harvest rate: $x(t)e_i(t)$, proportional to the current resource stock.

Resource Dynamics

$$\dot{x} = rx\left(1 - \frac{x}{K}\right) - x\left(e_1 + e_2\right)$$

logistic renewal – harvest

Revenue Redistribution

$$R_i = x\left[\left(1 - \frac{k}{2}\right)e_i + \frac{k}{2}e_j\right], k \in (-2,1)$$

Own extraction return + Opponent-linked share

$$R_i - xe_i = -\frac{k}{2}x(e_i - e_j)$$

$$R_i = (1-k)\,xe_i + \frac{k}{2}x(e_1 + e_2)$$

$k < 0$ winner-takes-more

$k = 0$ own-output benchmark

$k > 0$ pooling

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- Instantaneous payoff:

$$\pi_i(x, e_i, e_j; k) = x[(1 - \frac{k}{2})e_i + \frac{k}{2}e_j] - \frac{c}{2}e_i^2, \qquad j \neq i,\ c > 0.$$

- Discounted objective: player i chooses $e_i(\cdot) \geq 0$ to maximize

$$J_i\ (x_0; e_i, e_j) = \int_0^\infty e^{-\rho t}\pi_i\Big(x(t), e_i(t), e_j(t); k\Big)\ dt, \quad \rho > 0.$$

Feedback Nash Equilibrium

- Stationary state-feedback strategy: $\sigma_i : (0,K] \to [0,\infty)$, $\quad e_i(t) = \sigma_i(x(t))$.

DEFINITION — FEEDBACK NASH EQUILIBRIUM

A profile (σ_1^*, σ_2^*) is an FNE if

$$J_i(x_0; \sigma_i^*, \sigma_j^*) \geq J_i(x_0; u_i, \sigma_j^*),$$

for every player i, every $x_0 \in (0,K]$, and every admissible unilateral control u_i.

- Scope of analysis：
  - Symmetric equilibrium: $\sigma_1^* = \sigma_2^* = \sigma^*$.
  - Regular feedback: σ_i admits a nonnegative, continuous, piecewise-C^1 extension to $[0,K]$.

Existence

If $-2 < k \leq 1$ and $0 < \rho < r$, there exists a global continuous feedback law e^{sel} such that $(e^{\text{sel}}, e^{\text{sel}})$ is a symmetric stationary FNE.

Construction of FNE

$$\rho V(x) = \widehat{G}\left(x, V'(x); k\right)$$

A PROJECTED BEST RESPONSE

$$e\hat{}(x, p; k) = [\frac{x}{c}(1 - \frac{k}{2} - p)]_+$$

$$\widehat{G}(x, p; k) := x\hat{e} - \frac{c}{2}\hat{e}^2 + p[rx(1 - \frac{x}{K}) - 2x\hat{e}]$$

The lower bound e ≥ 0 creates two regimes.

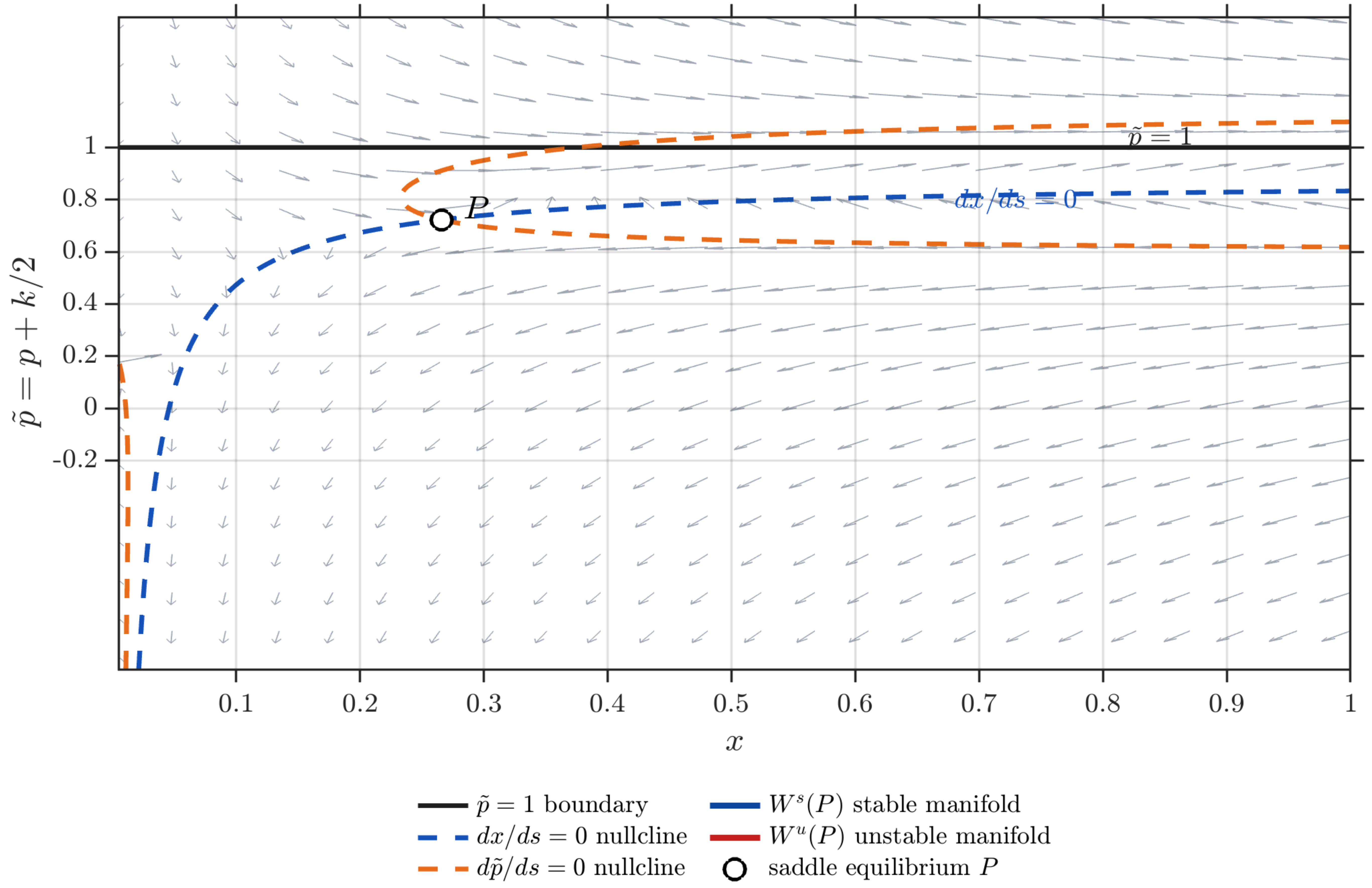
INACTIVE	ACTIVE
$e(x) = 0$	$e(x) > 0$
constraint binds	interior HJB branch
↓	↓
Closed-form equation	Auxiliary system representation

Auxiliary system representation

$$\frac{d\tilde{p}}{dx} = \Phi(x, \tilde{p}; k) := \frac{H(x, \tilde{p}; k)}{F(x, \tilde{p}; k)} \quad \leftrightarrow \quad \begin{cases} \dfrac{d\tilde{p}}{ds} = H(x, \tilde{p}; k) \\ \dfrac{dx}{ds} = F(x, \tilde{p}; k) \end{cases}$$

use phase-plane tools to identify solution branches

Construction of FNE



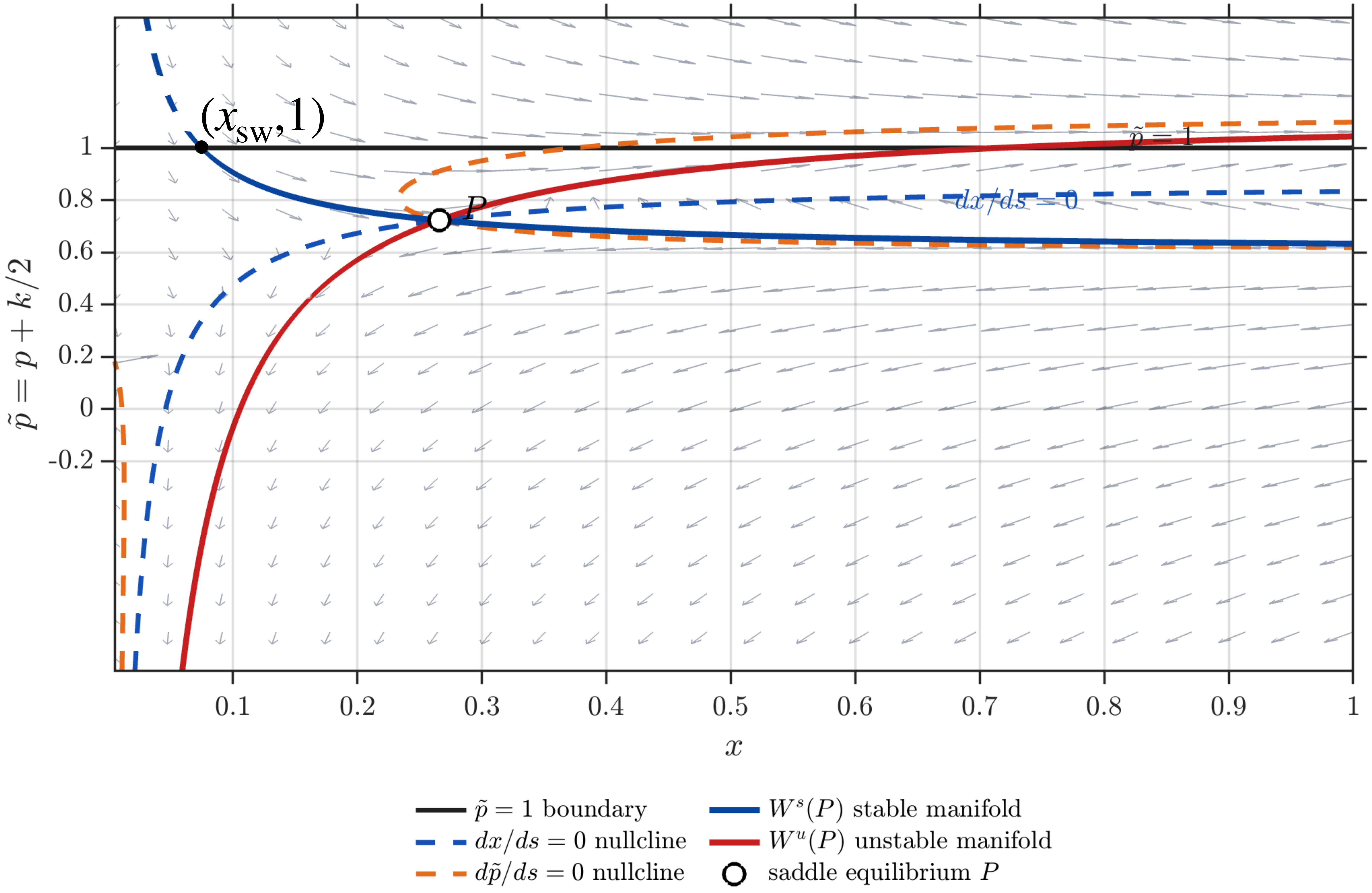
Phase portrait

For each fixed k, the active auxiliary system has the following structure:

- There exists a unique auxiliary equilibrium $P(k)$. Moreover, $P(k)$ is a hyperbolic saddle.
- The stable manifolds branch $\tilde{p}_s(x)$ is decreasing.
- The unstable manifolds branch $\tilde{p}_u(x)$ is increasing.

$$e^{\mathrm{sel}}(x) = \begin{cases} 0, & x \le x_{\mathrm{sw}}, \\ \dfrac{x}{c}\big(1 - \tilde{p}_s(x)\big), & x > x_{\mathrm{sw}}. \end{cases}$$

Construction of FNE



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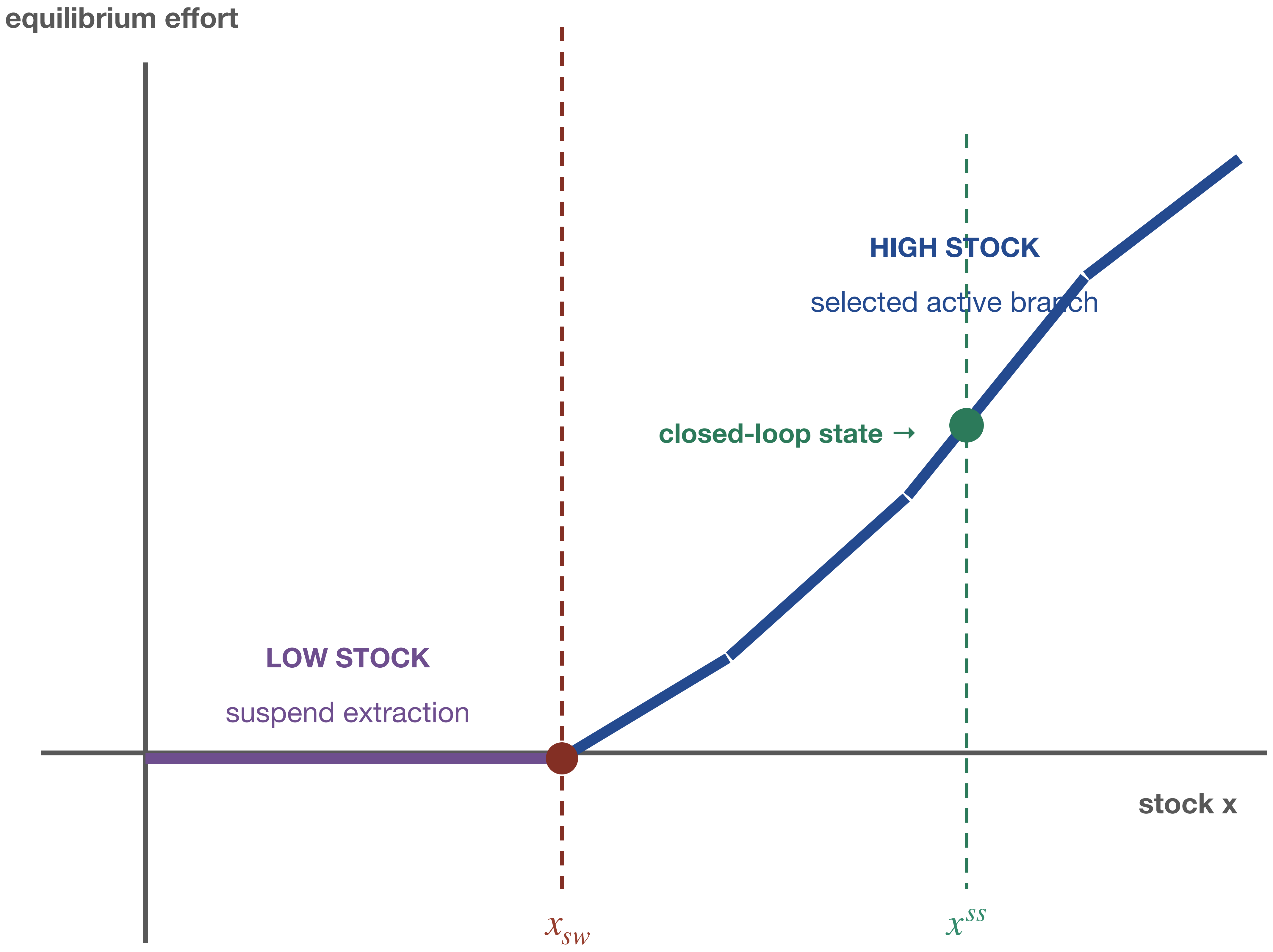
Construction of FNE

SELECTED FEEDBACK



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Uniqueness

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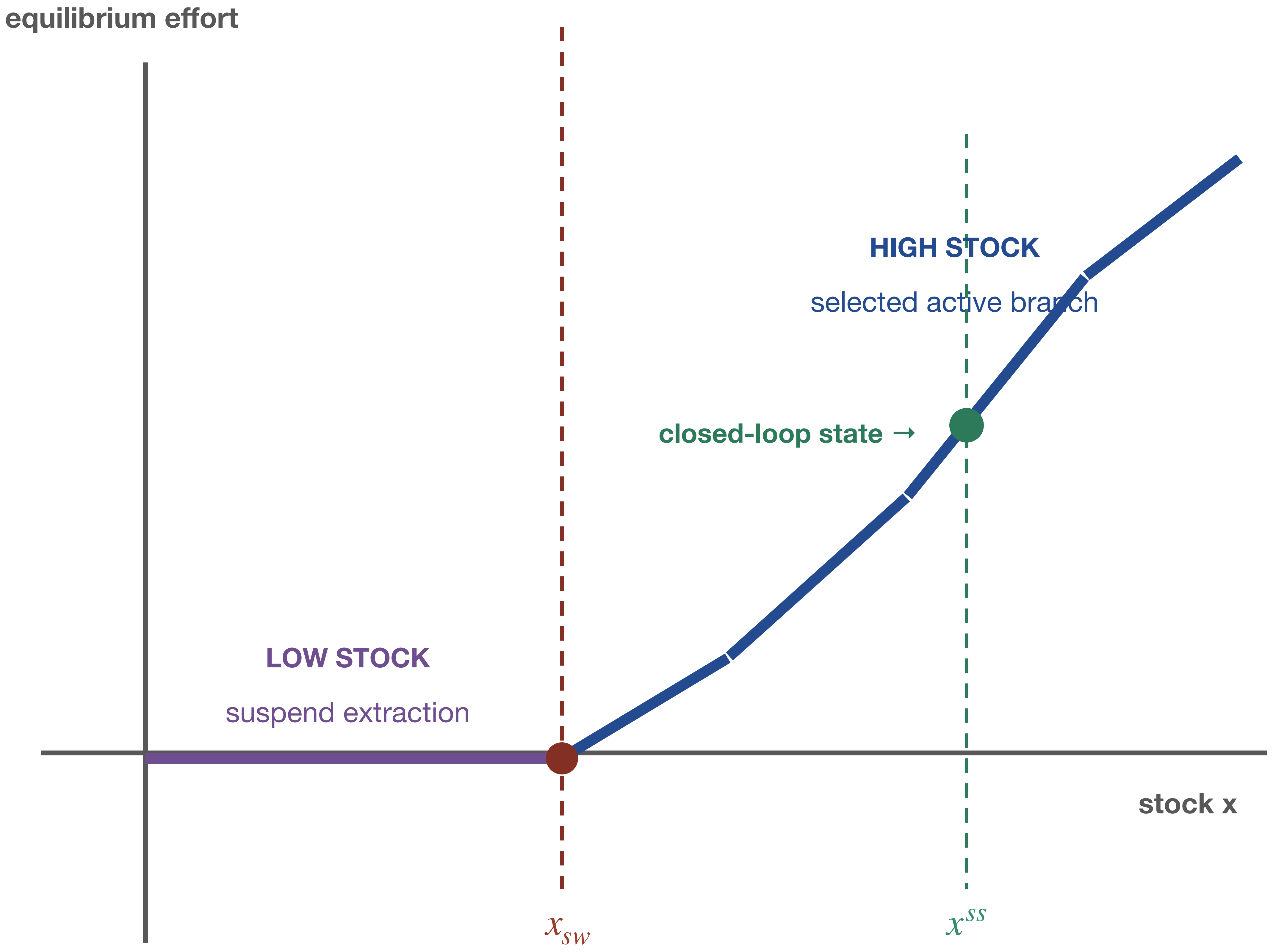

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Uniqueness

If $-2 < k \le 1$ and $0 < \rho < r$, the selected continuous symmetric stationary FNE is the unique symmetric stationary FNE in the regular Markov feedback class.

More pooling raises the selected steady stock

Dynamic under selected FNE

$$\dot{x} = rx(1 - \frac{x}{K}) - 2xe^{sel}(x)$$

$\Rightarrow$ The closed-loop trajectory converges to the unique steady state x^{ss}.

Monotonicity

Under $-2 < k \leq 1$ and $0 < \rho < r$, the equilibrium steady-state stock $x^{ss}(k)$ is strictly increasing in the redistribution parameter k:

$$\frac{dx^{ss}(k)}{dk} > 0.$$

Higher pooling raises the selected steady stock.

Three objectives imply three target-stock logics

STATIONARY NET PAYOFF

$$\Pi^{ss}(x) := xe^{ss}(x) - \frac{c}{2}\left(e^{ss}(x)\right)^2$$

Long-run incentive

INSTANTANEOUS PAYOFF

$$\Pi_{\text{in}}(k) := \pi\left(x_{\text{in}}, e^{\text{sel}}(x_{\text{in}}; k)\right).$$

Myopic incentive

DISCOUNTED GROSS OUTPUT

$$\mathcal{Y}_\delta(x_{\text{init}}; k) := \int_0^\infty e^{-\delta t}\, 2x_k(t)e^{\text{sel}}(x_k(t); k)\, dt$$

Social output incentive

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Social output incentive

$$sgn\left(\frac{d\Pi^{ss}}{dk}\right) = sgn(x_\Pi \dagger - x^{ss})$$

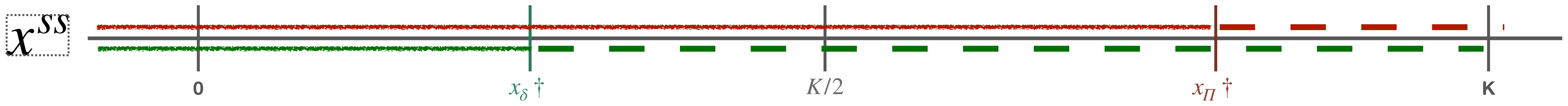
$$sgn(\partial_k \Pi_{in}) = sgn\left(e^{sel}(x_{in}; k) - \frac{x_{in}}{c}\right)$$

$$x_{in} \geq \min\{x^P(k), x^{ss}(k)\}$$

$$\max\{x_{\text{init}}, x^{\text{ss}}\} \leq x_\delta^\dagger \implies \frac{dY_\delta}{dk} \geq 0$$

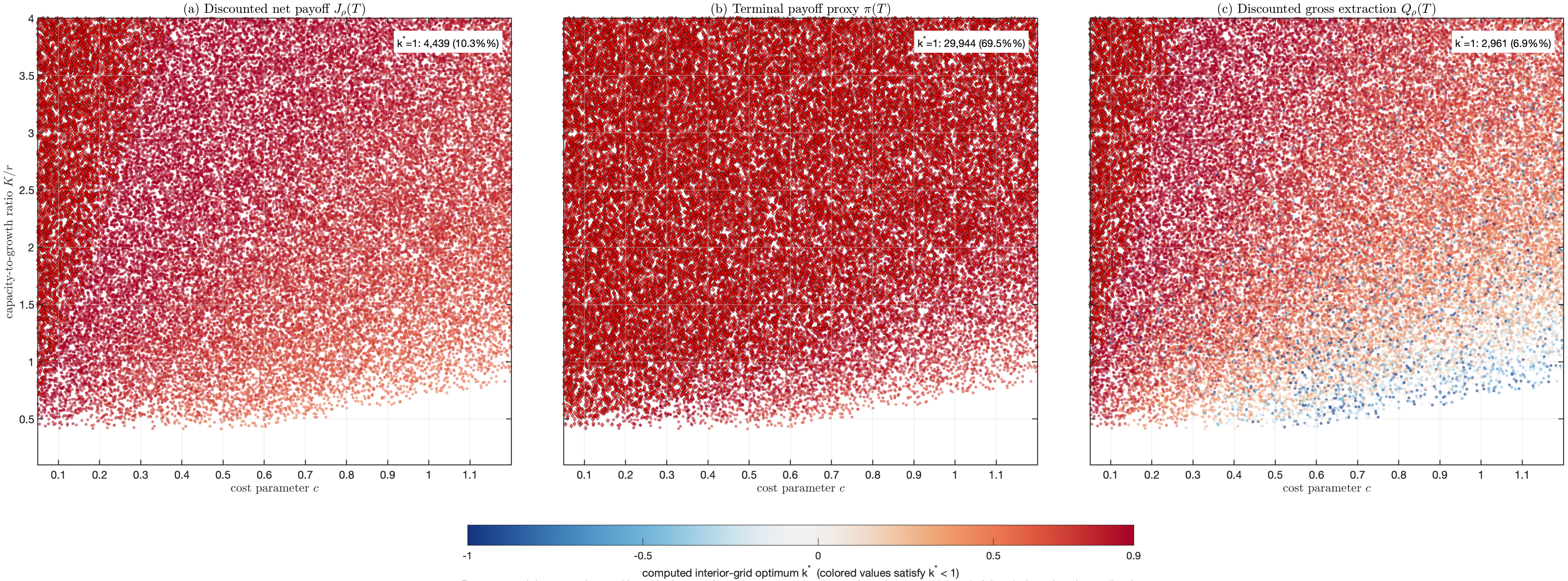
$$\min\{x_{\text{init}}, x^{\text{ss}}\} \geq x_\delta^\dagger \implies \frac{dY_\delta}{dk} \leq 0$$

$$x_\delta \dagger = \frac{K}{2}\left(1 - \frac{\delta}{r}\right) \qquad x_\Pi \dagger = \frac{K(2K + cr)}{4K + cr}$$



Computed policy regions depend on the evaluation criterion

$$k^*_{j,grid} = argmax_{k \in G} W_j(k)$$



Raw accepted draws are shown without binning; r and ρ vary across overlapping markers. Argmax grid: k = -1,-0.9,…,1; ties select the smallest k.

x_0=0.6 · 43,090 valid samples after K ≥ x_0 filtering · k=1 is a computational grid boundary outside the theorem interval k<1

Take-home messages

01 MECHANISM

Budget-balanced sharing changes resource dynamics through unilateral marginal incentives while preserving aggregate current gross revenue.

02 EQUILIBRIUM

Under the main regime, the unique regular symmetric stationary FNE has one inactive-to-active switching stock.

03 POLICY

Higher k raises the selected steady stock in the certified range, but different performance criteria prefer different stock levels.

Thank you for your attention.

Yi Han

PhD Candidate, Peking University, China.

han_yi@stu.pku.edu.cn